ON

THE NEW FORM

OF

THE ACHROMATIC OBJECT-GLASS

INTRODUCED BY STEINHEIL.

BY

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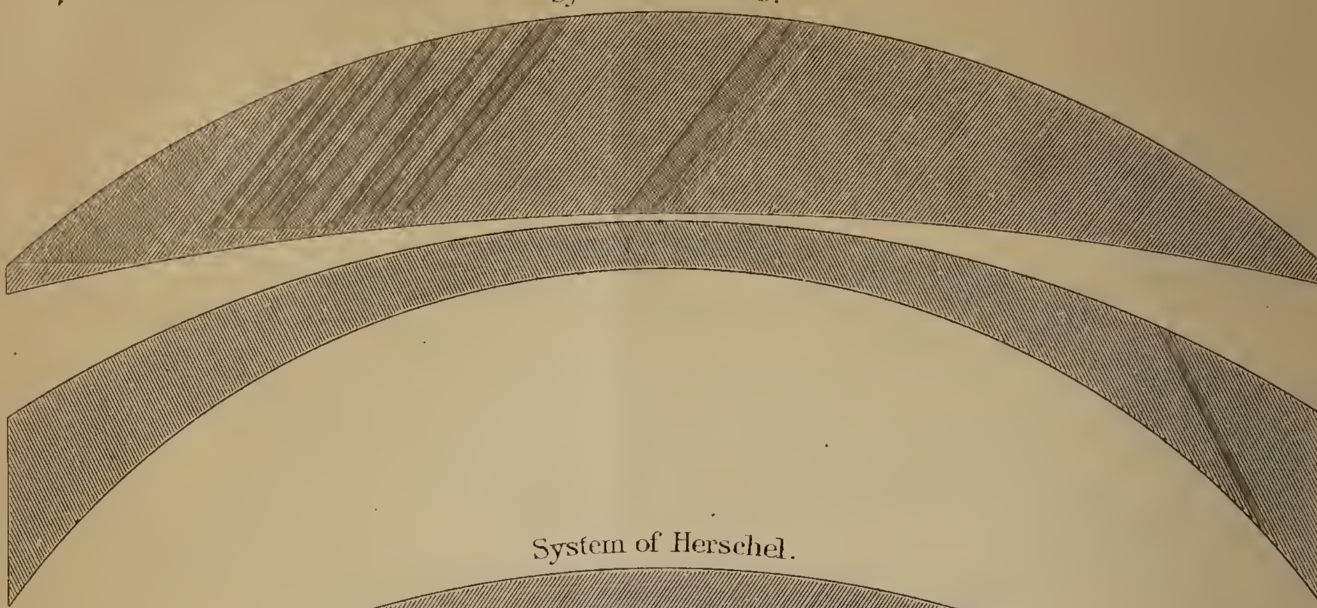
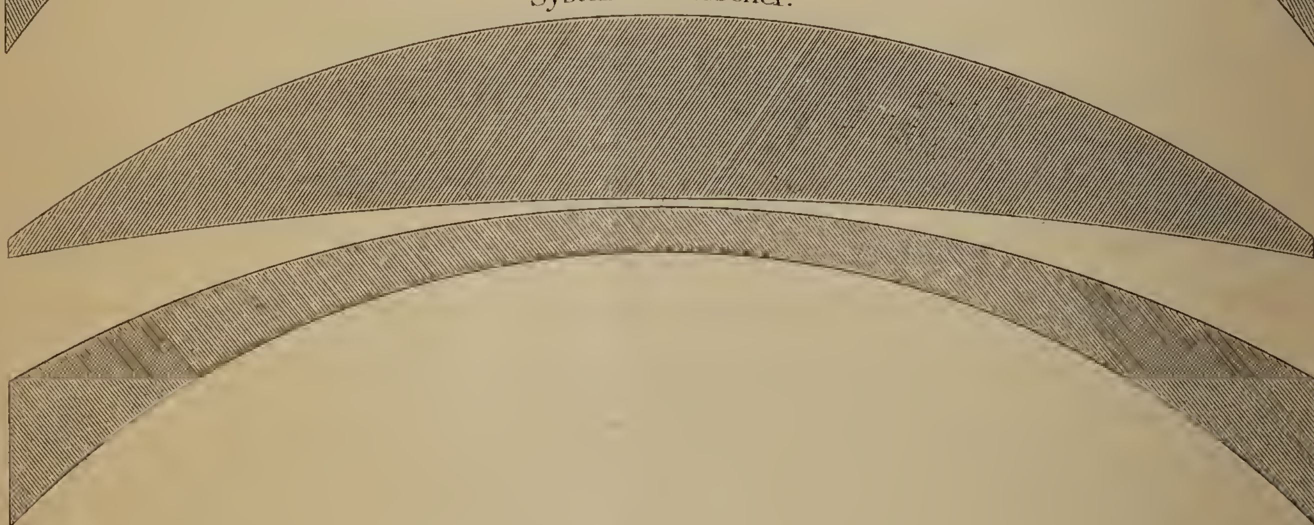
1863.

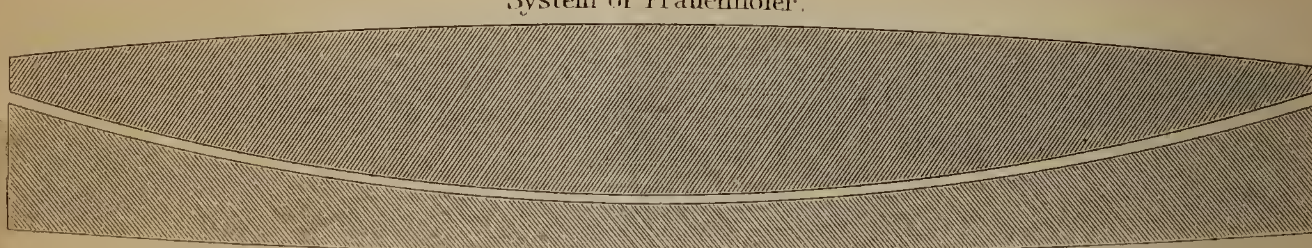
System of Gauss.



System of Herschel.



System of Frauenhofer.



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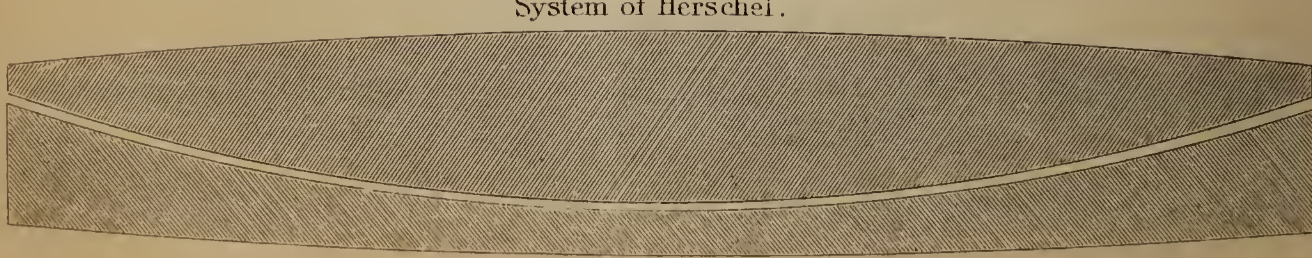

ON THE

NEW FORM OF THE ACHROMATIC OBJECT-GLASS.

In June, 1860, Professor Steinheil communicated to the Royal Academy of Sciences at Munich * a notice of an object-glass of thirty-six lines aperture, executed at his optical establishment according to the system of curves proposed by Gauss in an article published in the *Zeitschrift für Astronomie* of Lindenau and Bohnenberger, in 1817.†

This telescope, and subsequently another ‡ of similar form, but larger, have been carefully tested, and, in the opinion of competent judges, they have exhibited a more complete achromatism, and in other respects more perfect definition, than was to be found with object-glasses of the ordinary form, of equal dimensions.

Some part of this superiority may be attributable to the manner of mounting the lenses, which admits of readily changing their relative positions so as to effect the best adjustment by actual trial; a provision undoubtedly of considerable value, but perhaps equally applicable in the old system, if a slight separation of the inner surfaces of the crown and flint lenses were made one of the conditions for determining the curves. By this means, as Steinheil has remarked, we may not only diminish outstanding errors in the object-glass, but also, to some extent, the aberrations of the eyepiece, and even defects in the eye itself. There seems, however, to be no reason to doubt that these object-glasses owe their excellence mainly to the improved theory of their curves.

Among other advantages, the new combination admits of larger angles of aperture than would otherwise be practicable, without compromising the clearness of the definition. It is here, in fact, that the value of the improvement is best illustrated. Any shortening of the focal length accomplished without sacrificing illuminating power, or

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* Sitzungsberichte der königl. bayer. Akademie der Wissenschaften zu München, 1860, II. 160.

† Lindenau und Bohnenberger, Zeitschrift für Astronomie, Nov., Dec. 1817, IV. 345.

‡ Sitzungsberichte, 1860, V. 662.

defining qualities, is a substantial gain in more than one direction. It reduces the telescope to a more manageable size, which, in one of the larger class, is a matter of the first importance, for not only is the size of the dome and building required to protect it, and, in general, the cost of all the accessory apparatus necessary for its efficiency, largely diminished by a reduction in the length of the focus, but the facility of using it depends also very much on the same condition. Again, by shortening the tube, we apply the best means of reducing its flexure, — one of the most intractable of all sources of error in meridian instruments.

Hitherto, in the practice of the best opticians, the apertures of the largest object-glasses have not exceeded $\frac{1}{15}$ of the focal length, which is the proportion in Mr. Clark's 18.5-inch lens. With those of moderate size the ratio of $\frac{1}{13}$ to $\frac{1}{14}$ has been successfully employed. Of such, the object-glasses of the vertical circle and prime vertical instrument at Poulkova, of 6-inch aperture, are examples of remarkable excellence. At present, however, Messrs. Merz are prepared to extend the ratio of $\frac{1}{14}$ even to lenses of $19\frac{1}{4}$ inches (English) aperture; a gain in the surface exposed to the light for the same focal length, of nearly seventy per cent. Steinheil has stated, that, with the Gaussian objectives, ratios of the aperture to the focal length as large as $\frac{1}{10}$ can be used for the largest refractors.* It must be remembered, that, owing to the strong curvature of the surfaces, the light has to traverse a greater thickness of the glass, and must experience more than ordinary loss from extinction. Perhaps, also, there will be a sensibly greater loss from reflection, from the greater inclination of the incident ray to the surface near the margin. The gain in area will, therefore, not represent precisely the increase in illuminating power.

There are two other objections to the new construction which may be thought in some measure to counterbalance its special advantages: one of these is the much greater depth of its curves, suggesting, perhaps without sufficient foundation, practical difficulties of workmanship. That they have been actually overcome in lenses of moderate size is certainly the best reason for anticipating success when the trial is made on a larger scale. It is further evident, from the peculiar form of the lenses, each of which is a meniscus, that, if they are worked out of flat discs, as usual, greater thickness of material will be required. This

* Sitzungsberichte der königl. bayer. Akad. der Wiss., 1860, V. 663.

would increase the difficulty, already so great, of procuring suitable glass. It is possible that the material could be accommodated nearly to the ultimate form of the lenses, just as, in the present process of manufacture, an irregular mass is moulded into a flat disc, approximating to the shape required. It does not appear that either of these obstacles would long remain in the way of the general adoption of the new system, if its advantages were distinctly recognized, and sufficient inducements were offered to artists and to the manufacturers of optical glass to turn their efforts in this direction.

The contrast presented in the character of the curves in the two combinations, which is so decided that the eye at once distinguishes between them without any occasion for measurement or exact comparison, is very remarkable; for if the superiority of Gauss's combination be admitted, it shows that the practice of opticians has been confined to a region altogether removed from that in which the best system is to be found. In this they have only adopted the recommendations of the many eminent mathematicians who have treated of the theory of the achromatic object-glass.

The question proposed in this theory is to ascertain that form and disposition of the surfaces of two or more lenses, composed of materials of different dispersive powers, which shall most effectually destroy the aberrations of color and of figure. The problem, in the form in which it has been practically presented, is indeterminate, so that, for instance, in the case of lenses of crown and flint glass, "For every lens of crown-glass of positive focus, whatever the radii of its surfaces may be, a lens of flint-glass can be computed which will form, when united with it, an achromatic object-glass," — achromatic, that is to say, in the limited sense in which the term is commonly accepted.

This allows, of course, of a great range in the choice of curves, and a variety of conditions have been proposed for determining the selection. In one respect only has there been a general consent of authorities. The front lens has always been convex on both surfaces. But it would seem that in this particular the direction given to the investigation has not been fortunate. It is at least an oversight, that the relative importance of the two principal sources of indistinctness has not been kept prominently in view. For while it is admitted that the chromatic dispersion is the chief source of indistinctness, the arbitrary condition has not been determined with special reference to this circumstance.

This omission has been supplied by Gauss, who has given attention mainly to the more complete elimination of the aberration of color, while, at the same time, his expectations that this could be done without sensibly increasing the spherical aberration, have been fully realized in the performance of the new object-glasses. Indeed, it deserves notice that the resulting curves bear a considerable resemblance to one of the systems which has been designed with express reference to the correction of the spherical aberration. Allusion is here made to the forms deduced by Herschel * for the elimination of the spherical aberration of diverging, as well as parallel rays. From the comparisons subjoined, it will be seen that one of the solutions satisfying his equations approximates nearly to Gauss's system, while the other approaches to a form employed by Frauenhofer. So far, therefore, as this holds good, each fulfils the conditions proposed in Herschel's theory.

As Gauss has published neither the mathematical investigation of the subject, nor even the final equations from which his curves were computed, we have not the means of deciding with entire certainty, whether the resemblance referred to is merely accidental, or whether it expresses an affinity involved in the nature of the problem. But the latter seems the more probable explanation. The numerical values of the radii in his system, computed for a special case, are here transcribed from his original memoir,† after reducing them to a focal length, for the two lenses combined, of twenty-one French feet, for the sake of comparison with the large Munich refractors.

I. *Gauss's Curves.*

						ft.
1st surface	of the	crown lens,	convex,	radius	=	+ 2.535
2d "	"	"	concave,	"	=	— 7.521
1st "	"	flint lens,	convex,	"	=	+ 3.123
2d "	"	"	concave,	"	=	— 2.084 ‡
		Compound focus,			=	21.00

In No. 1289 of the *Astronomische Nachrichten* Oudemans has given the following measurements of an object-glass made by Frauenhofer for the Equatorial of the Observatory at Utrecht. The numbers have been reduced to the same unit as before, assuming the focal length from *Astr. Nach.* 1281.

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* Phil. Trans., 1821, p. 258.

† Zeitschrift für Astron., IV. 350.

‡ This number has been corrected to accord with the *erratum* noticed at the end of the volume cited.

II. *Frauenhofer's Curves.*

							ft.
1st surface of the crown lens, convex, radius,						=	+ 14.157
2d	"	"	"	"	"	=	+ 5.635
1st	"	"	flint lens,	concave,	"	=	— 5.775
2d	"	"	"	convex,	"	=	+ 25.945
			Compound focus,			=	21.00

Another of his object-glasses, probably computed from a similar formula, but for glass of slightly different refractive and dispersive powers, has values of the radii as follows*: —

III. *Frauenhofer's Curves.*

							ft.
1st surface of the crown lens, convex, radius,						=	+ 15.430
2d	"	"	"	"	"	=	+ 6.144
1st	"	"	flint lens,	concave,	"	=	— 6.262
2d	"	"	"	convex,	"	=	+ 22.461
			Compound focus,			=	21.00

These numbers we will now compare with the two solutions of Herschel's equations, using the notation l, r and r', to denote the reciprocals of the compound focal length and of the radii of the front surfaces of the two lenses. The substitution of the values of the indices of refraction and of the dispersive powers which have been used by Gauss for computing the system I. gives the relations †: —

$$0 = 2.3200\, r^2 - 21.31\, l r + 59.57\, l^2 + 3.5792\, l r' - 1.4233\, r'^2$$
$$0 = 6.6400\, r - 24.95\, l - 4.1119\, r'$$

From which we have

$$0 = -1.3917\, r^2 + 12.37\, l r - 14.56\, l^2$$

with the roots

$$\frac{r}{l} = 7.4922, \quad \text{and} \quad \frac{r}{l} = 1.3964,$$

which afford the subjoined two sets of values.

IV. *Herschel's Curves.*

$$\frac{r}{l} = 7.4922$$

							ft.
1st surface of the crown lens, convex, radius,						=	+ 2.803
2d	"	"	"	concave,	"	=	— 9.525
1st	"	"	flint lens,	convex,	"	=	+ 3.482
2d	"	"	"	concave,	"	=	— 2.361
			Compound focus,		"	=	21.00

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* Zeitschrift für Astron., IV. 352.

† In the equation (*z*) Phil. Trans. 1821, p. 258, the coefficient of ϖ^2 has been corrected from $\frac{2\mu' + 1}{\mu' - 1}$ to $\frac{3\mu' + 1}{\mu' - 1}$. *Vide* Article on Light, Encyc. Met., p. 424.

V. $\frac{r}{l} = 1.3964$

							ft.
1st surface of the crown lens,				convex,	radius,	=	+ 15.038
2d	"	"	"	convex	"	=	+ 5.397
1st	"	"	flint lens,	concave,	"	=	— 5.507
2d	"	"	"	convex,	"	=	+ 22.135
			Compound focus,			=	21.00

With radii proportional to these numbers the figures in the accompanying Plate have been constructed, representing sections of the different object-glasses, each having a focal length of two feet, and an aperture of nearly four inches. The ratio of the aperture to the focal length has been taken larger than can be adopted in practice, in order to exaggerate the amount of curvature. It will be seen that the curves in the systems of Gauss and Frauenhofer may be nearly represented by the two solutions of Herschel's equations.*

It follows that Gauss's form, originally designed to secure a more complete elimination of the chromatic dispersion, must be also rather favorable than otherwise as regards the correction of the aberration of figure. It may be remarked, further, that his investigation, neglecting the thickness and distance of the lenses, leads to an equation of the fourth degree, which has no solution corresponding to V., nor to the above values of the radii used by Frauenhofer. On the other hand, if the curves in III. and V. have been derived from substantially the same theory, which seems a probable inference, it is scarcely possible that Frauenhofer should not have had at some time under consideration the system represented by the other solution of the equations, which would have conducted to forms approximating very nearly to the system of Gauss.

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* The refractive and dispersive powers in III., and probably in II., differ by small amounts from those used in computing IV. and V.; moreover, in the latter, the effect of the thickness of the lenses and of their distance from each other has not been included, so that the numbers to be strictly comparable would require a small correction. The values V., computed with the elements of refraction and dispersion used for III., neglecting only the correction for thickness, become

ft.
+ 14.212
+ 6.349
— 6.488
+ 25.375
21.00